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Translated by L.K.

PMM U.S.S.R.,Vol.49,No.6,pp.777-782,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

ON A METHOD OF SOLVING TWO-DIMENSIONAL INTEGRAL EQUATIONS OF AXISYMMETRIC CONTACT PROBLEMS FOR BODIES WITH COMPLEX RHEOLOGY*

A.V. MANZHIROV

A two-dimensional integral equatin appearing in axisymmetric contact problems for bodies with complex rheology is studied. A method of constructing the solution of this equation in proposed, based on inspecting the non-classical spectral properties of an integral operator. A contact problem for a non-uniformly aging viscoelastic foundation is solved as an example.

1. Consider the integral equation

$$c(t)(\mathbf{I}-\mathbf{L}_1)q(r,t)+(\mathbf{I}-\mathbf{L}_2)\mathbf{F}q(r,t)=\delta(t)-g(r) \tag{1.1}$$

$$(\mathbf{I}-\mathbf{L}_k)f(t)=f(t)-\int_1^t f(\tau)K_k(t,\tau)d\tau \quad (k=1,2) \qquad \mathbf{F}v(r)=\int_\varepsilon^1 v(\rho)k(\rho,r)\rho\,d\rho, \quad 0\leqslant\varepsilon<1$$

$$c(t)>0,\ \delta(t)\in C[1,T];\ g(r)\in L_2(\Omega);\quad q(r,t)\in L_2(\Omega)\times C[1,T]$$

with the auxilliary condition

$$P(t)=\int_\varepsilon^1 q(r,t)r\,dr,\quad P(t)\in C[1,T] \tag{1.2}$$

Here $K_k(t,\tau)$ are Volterra kernels /1/, the operator $\mathbf{F}$ is completely continuous, selfconjugate and positive definite and acts from $L_2(\Omega)$ into $L_2(\Omega)$, Ω is a region bounded by circles of radii ε and 1 (when $\varepsilon=0$ Ω is a unit circle), and

$$\int_\varepsilon^1\int_\varepsilon^1 k^2(\rho,r)r\rho\,dr\,d\rho<\infty \tag{1.3}$$

Note that the kernel of the integral operator $\mathbf{F}$ admits of the representation /2/

$$k(r,\rho)=\sum_{m=0}^\infty\sum_{n=0}^\infty r_{mn}P_m^*(r)P_n^*(\rho) \tag{1.4}$$

where $P_m^*(r)$ is a complete system of functions orthonormalized in $L_2(\Omega)$. We choose this function as follows ($P_m(x)$ is the Legendre polynomial):

$$P_m^*(r)=\sqrt{\frac{4m+2}{1-\varepsilon^2}}\,P_m\left(\frac{\varepsilon^2+1-2r^2}{1-\varepsilon^2}\right)\quad (m=0,1,2,\ldots),\qquad \int_\varepsilon^1 P_m^*(r)r\,dr=\begin{cases}[(1-\varepsilon^2)/2]^{1/2}, & m=0\\ 0, & m\neq 0\end{cases} \tag{1.5}$$

Moreover, by virtue of (1.3) and Parseval's equation, we have

$$\sum_{m=0}^\infty\sum_{n=0}^\infty r_{mn}^2<\infty \tag{1.6}$$

2. Let us determine in (1.1), (1.2) $q(r,t)$ and $\delta(t)$, assuming that the remaining functions are given.

We introduce a space of functions belonging to $L_2(\Omega)$ and such that their integral over Ω is zero, and denote it by $L_2^c(\Omega)$.

*Prikl.Matem.Mekhan.,49,6,1019-1025,1985

Theorem 1. The space $L_2^\circ(\Omega)$ is complete Hilbert space any function of which can be represented by a series over the orthonormalized system of functions (1.5), beginning with the first.

Let us consider the fundamental sequence $\{f_n\} \in L_2^\circ(\Omega)$. Since $L_2^\circ(\Omega) \subset L_2(\Omega)$, the sequence $\{f_n\}$ converges at least to $f \in L_2(\Omega)$, i.e.

$$\| f - f_n \|_{L_2(\Omega)} < \varepsilon_0, \quad n > N \tag{2.1}$$

where ε_0 is an arbitrarily small, previously specified positive number. We shall show that $f \in L_2^\circ(\Omega)$. We have

$$\left| \int_\Omega f\, d\omega \right| = \left| \int_\Omega (f - f_n)\, d\omega + \int_\Omega f_n\, d\omega \right| \leqslant \tag{2.2}$$

$$\int_\Omega |f - f_n|\, d\omega \leqslant \sqrt{\pi(1-\varepsilon^2)}\, \| f - f_n \|_{L_2(\Omega)}$$

But f is independent of n, therefore taking into account (2.1), (2.2) we obtain

$$\int_\Omega f\, d\omega = 0, \quad f \in L_2^\circ(\Omega)$$

The assertion concerning the representation of the functions $L_2^\circ(\Omega)$ follows from the properties of $P_m^*(r)$ (see (1.5)).

Theorem 2. The kernel $k(\rho, r)$ can be written in the form

$$k(\rho, r) = k^\circ(\rho, r) + [2/(1-\varepsilon^2)]^{1/2} k^1(r) + [2/(1-\varepsilon^2)]^{1/2} k^1(\rho) + [2/(1-\varepsilon^2)]\, D$$

$$k^\circ(\rho, r) = k^\circ(r, \rho), \quad \int_\varepsilon^1 k^\circ(\rho, r)\, \rho\, d\rho = 0$$

$$k^1(r) \in L_2^\circ(\Omega),\ D = \text{const}$$

The proof of the theorem can be constructed using expansion (1.4), relations (1.5), Perseval's equation, estimate (1.6) and Theorem 1.

We shall merely note that

$$k^\circ(\rho, r) = \sum_{m=1}^\infty \sum_{n=1}^\infty r_{mn} P_m^*(\rho) P_n^*(r) \tag{2.3}$$

$$k^1(r) = \sum_{n=1}^\infty r_{0n} P_n^*(r), \quad D = r_{00}$$

Theorem 3. The operator

$$\mathbf{F}^\circ : \mathbf{F}^\circ f(r) = \int_\varepsilon^1 f(\rho)\, k^\circ(\rho, r)\, \rho\, d\rho$$

is completely continuous, selfconjugate and positive definite, and acts from $L_2^\circ(\Omega)$ into $L_2^\circ(\Omega)$.

The operator $\mathbf{F}^\circ$ is selfconjugate by virtue of (2.3), and its complete continuity follows from the estimate (see (1.6))

$$\int_\varepsilon^1 \int_\varepsilon^1 [k^\circ(\rho, r)]^2 \rho r\, d\rho\, dr = \sum_{m=1}^\infty \sum_{n=1}^\infty r_{mn}^2 < \infty$$

while the positive definiteness follows from the relation

$$(\mathbf{F}^\circ f, f)_{L_2^\circ(\Omega)} = (\mathbf{F} f, f)_{L_2(\Omega)}, \quad f \in L_2^\circ(\Omega)$$

Theorem 4. The sequence of eigenfunctions φ_i° corresponding to the eigenvalues α_i° $(i = 1, 2, \ldots)$ of the operator $\mathbf{F}^\circ$, forms an orthonormalized basis in $L_2^\circ(\Omega)$.

The theorem is a corollary of the general theory of operators in Hilbert spaces /3/.

Theorem 5. The sequence $\{\varphi_i^\circ\}$ $(i = 0, 1, 2, \ldots;\ \varphi_0^\circ = [2/(1-\varepsilon^2)]^{1/2})$ forms an orthonormalized basis in $L_2(\Omega)$.

We have

$$f(r) = \sum_{i=0}^\infty f_i P_i^*(r) = f_0 P_0^*(r) + \sum_{i=1}^\infty f_i P_i^*(r) \tag{2.4}$$

In accordance with Theorems 1, 4 and (2.4),

$$\sum_{i=1}^\infty f_i P_i^*(r) = \sum_{i=1}^\infty f_i^\circ \varphi_i^\circ(r) \in L_2^\circ(\Omega), \qquad f(r) = \sum_{i=0}^\infty f_i^\circ \varphi_i^\circ(r), \quad \varphi_0^\circ(r) = P_0^*(r), \quad f_0^\circ = f_0$$

which it was required to prove.

Let us write the solution in the form

$$q(r,t)=\sum_{i=0}^{\infty} z_i(t)\,\varphi_i^\circ(r) \tag{2.5}$$

$$g(r)=\sum_{i=0}^{\infty} g_i^\circ\varphi_i^\circ(r),\quad k^1(r)=\sum_{i=1}^{\infty} k_i\varphi_i^\circ(r)$$

Substituting (2.5) into (1.1), (1.2) and taking the theorems 1—5 into account, we obtain

$$z_i(t)=f_i(t)+\int_1^t f_i(\tau)\,R_i^\circ(t,\tau)\,d\tau \quad (i=1,2,\ldots)$$

$$f_i(t)=-[k_i(\mathbf{I}-\mathbf{L}_2)\,z_0(t)+g_i^\circ]/[c(t)+\alpha_i^\circ]$$

$$\delta(t)=[2/(1-\varepsilon^2)]^{1/2}[c(t)(\mathbf{I}-\mathbf{L}_1)\,z_0(t)+$$

$$D(\mathbf{I}-\mathbf{L}_2)\,z_0(t)+(\mathbf{I}-\mathbf{L}_2)\sum_{i=1}^{\infty} z_i(t)\,k_i+g_0^\circ]$$

$$z_0(t)=[2/(1-\varepsilon^2)]^{1/2}P(t)$$

where $R_i^\circ(t,\tau)$ is the resolvent of the kernel

$$K_i^\circ(t,\tau)=[c(t)\;K_1(t,\tau)+\alpha_i^\circ K_2(t,\tau)]/[c(t)+\alpha_i^\circ]$$

3. Let us now assume that all functions except $q(r,t)$ and $P(t)$ are given. We will determine the remaining functions using the classical method of the spectral theory of operators /1—3/.

Let us write the solution in the form

$$q(r,t)=\sum_{i=0}^{\infty} \omega_i(t)\,\varphi_i(r) \tag{3.1}$$

$$g(r)=\sum_{i=0}^{\infty} g_i\varphi_i(r),\quad 1=\sum_{i=0}^{\infty} \delta_i\varphi_i(r)$$

where $\varphi_i(r)$ $(i=0,1,2,\ldots)$ are eigenfunctions of the operator $\mathbf{F}$ corresponding to its eigenvalues α_i.

Substituting (3.1) into (1.1), (1.2) we obtain

$$\omega_i(t)=\psi_i(t)+\int_1^t \psi_i(\tau)\,R_i(t,\tau)\,d\tau$$

$$\psi_i(t)=\frac{\delta_i\delta(t)-g_i}{c(t)+\alpha_i},\qquad P(t)=\sum_{i=0}^{\infty}\omega_i(t)\,\delta_i$$

where $R_i(t,\tau)$ is the resolvent of the kernel

$$K_i(t,\tau)=[c(t)\;K_1(t,\tau)+\alpha_i\,K_2(t,\tau)]/[c(t)+\alpha_i]$$

We can prove the following theorem.

Theorem 6. The solution of (1.1) with condition (1.2) exists in the chosen class of functions, is unique and can be found with prescribed accuract using the methods given.

Unlike the well-known methods of representing the solution in the form of a series over an orthogonal system of pclynomials or eigenfunctions of the operator $\mathbf{F}$, the method of Sect.2 enables us to avoid the need to solve an infinite system of integral Volterra equations and to satisfy the auxilliary condition exactly. When the auxilliary condition (1.2) is given, the method of Sect.2 enables us to construct a solution of (1.1) (when $t=1$, it becomes a Fredholm equation of the second kind) on the spectrum of the operator $\mathbf{F}$ (at least in the case when all its eigenvalues are simple). The last assertion follows from the formulas of Sect.2 where the expansions of the solution contain only α_i° different from the eigenvalues α_i of the operator $\mathbf{F}$. When the right-hand side of Eq.(1.1) is given, we cannot construct a solution in the spectrum of $\mathbf{F}$, which is in complete accord with the theory of integral equations (see the formulas of Sect.3).

When the algorithms of Sect.1 and 2 are realized directly, it is convenient to write the eigenfunctions in the form of a series over the system of polynomials (1.5), followed by their determination using the Bubnov-Galerkin method /4/.

It should be noted that related equations appeared in /5/.

4. Let us now consider applications. In the creep theory of non-uniformly aging bodies /6, 7/ a number of axisymmetric contact problems lead to the integral equation (1.1) with

auxilliary condition (1.2). We shall consider one of them.

Let us assume that a rigid stamp is impressed without friction by a force $P(t)$ into a two-layer foundation consisting of a non-uniformly aging thin layer /8/ and a uniformly aging layer of any thickness H. The region of contact does not vary with time, and is bounded by circles of radii b and a $(a > b)$ and theform of the stamp foundation is a function of $g(r)$. The thin layer of thickness h lies frictionlessly on the uniformly aging layer, the latter, in turn, being at rest on the non-deformable base. Using the results of /8–11/ and taking into account the change of variables, we have

$$\begin{aligned}
&t^* = t\tau_1^{-1},\ \tau^* = \tau\tau_1^{-1},\ r^* = ra^{-1},\ \varepsilon = ba^{-1} \qquad (4.1)\\
&\varkappa^*(z) = \varkappa(z)\,\tau_1^{-1},\ c(t) = 0{,}5\ ha^{-1}\,\theta_2(t-\tau_2)\,\theta_1^{-1}(t)\\
&q^*(r^*, t^*) = q(r, t)\,\theta_2^{-1}(t-\tau_2),\ \delta^*(t^*) = \delta(t)\,a^{-1}\\
&g^*(r^*) = g(r)\,a^{-1},\ \lambda = Ha^{-1},\ k^*(\rho^*, r^*) =\\
&\quad k(rH^{-1}, \rho H^{-1})\,\lambda^{-1},\ P^*(t^*) = [2\pi a^2\theta_2(t-\tau_2)]^{-1}P(t)\\
&K_2^*(t^*, \tau^*) = K_2(t-\tau_2, \tau-\tau_2)\,\tau_1,\ K_1^*(t^*, \tau^*) =\\
&\quad \theta_2(\tau-\tau_2)\,\theta_1(t)\,\theta_2^{-1}(t-\tau_2)\,\theta_1^{-1}(\tau)\,\tau_1 K_1^1(t, \tau)\\
&K_1^1(t,\tau) = h^{-1}\int_0^h K_1(t+\varkappa(z), \tau+\varkappa(z))\,dz\\
&K_i(t,\tau) = E_i(\tau)\frac{\partial}{\partial\tau}\left[\frac{1}{E_i(\tau)} + C_i(t,\tau)\right] \quad (i = 1, 2)\\
&[E_1^1(t)]^{-1} = h^{-1}\int_0^h E_1^{-1}(t+\varkappa(z))\,dz\\
&\theta_1(t) = 0{,}5\ E_1^1(t)/(1-\nu_1^2),\ \theta_2(t) = 0{,}5\ E_2(t)/(1-\nu_2^2)
\end{aligned}$$

(t is the time, τ is the integration variable in real time, r is the radial coordinate, $\varkappa(z)$ is a function of non-uniform aging across the depth of the thin layer, τ_1 is the increase in the elements of its lower face at the instant the load is applied, ρ is the variable of integration in the real length domain, $q(r,t)$ is a function of the contact pressures, $\delta(t)$ is the subsidence function, $K_i(t,\tau)$, $C_i(t,\tau)$, $E_i(t)$, ν_i are the creep kernels and measures, moduli of instantaneous elastic deformation and Poisson's ratios of the upper $(i = 1)$ and lower $(i = 2)$ layer, τ_2 is the instant of establishing the lower layer, and $k(rH^{-1}, \rho H^{-1})$ is the Fredholm kernel of the contact problem), and omitting the asterisks in the notation, we arrive at the initial problem of Sect.2. It should be noted that for the non-uniformly aging viscoelastic layered foundations /8, 11/ the problems of impressing circular and annular stamps must lead to the solution (1.1), (1.2).

To illustrate the method, we will solve the contact problem formulated under the assumption that the packet of layers is made of concrete, and its lower face is connected to a rigid foundation. Assuming that the elastic characteristics are contact, we take the following values of the functions and parameters:

$$\begin{aligned}
&C(t,\tau) = \varphi(\tau)\,f(t-\tau),\quad \varphi(\tau) = C_0 + Ae^{-\beta_1\tau},\quad \beta = \beta_1\tau_1\\
&f(t-\tau) = (1-e^{-\gamma_1(t-\tau)}),\quad \lambda = \lambda_1\tau_1,\quad g(r) = 0,\quad \varepsilon = 0\\
&c(t) = 0.2,\quad \lambda = 2,\quad C_0E = 0.5522,\quad AE = 4\\
&\tau_2 = 0,\quad \nu = 0.3,\quad \beta_1 = 0.031\,\text{days}^{-1}, \gamma_1 = 0.06\,\text{days}^{-1}.
\end{aligned}$$

Note that when the creep measure is given in the form /10/ the functions $z_i(t)$ and $\omega_i(t)$ can be found in explicit form using the resolvent of the Arutyunyan kernel. The form of the kernel of the contact problem with coupling along the lower face of the foundation, and a formula for the coefficients of its expansion were obtained in /11/.

Let us consider the case of natural and artificial non-uniform aging of the foundation, introducing the parameter of non-uniform aging /11/ (below, we shall use the dimensionless values of the parameters in accordance with (4.1)

$$\mu = h^{-1}\int_0^h e^{-\beta\varkappa(z)}\,dz$$

1. Natural non-uniform aging (the growth of the elements of the upper layer decreases with height)

$$1 \leqslant \mu < e^{\beta},\quad \tau_1 = 100\,\text{days}\ P(t) = 1 - e^{10(1-t)}$$

2. Artificial non-uniform aging (the growth of the elements of the upper later increases with height

$$0 < \mu \leqslant 1,\quad \tau_1 = 10\,\text{days}\ P(t) = 1 - e^{(1-t)}$$

We note that both versions include, as limiting case, the cases of uniform aging of the packet $(\mu = 1)$ characterized by its growth at the instant τ_1 the load is applied.

We show in the graphs below the relations for $\mu = 1$ (uniform aging) (the solid lines), for $\mu = 10$ (natural non-uniform aging) (the dashed lines) and for $\mu = 0,1$ (artificial non-uniform aging) (the dash-dot lines). The lower scale of dimensionless time will always correspond to the dashed lines, and the upper scale to the dash-dot lines.

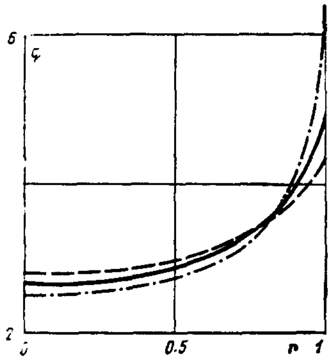

Fig.1

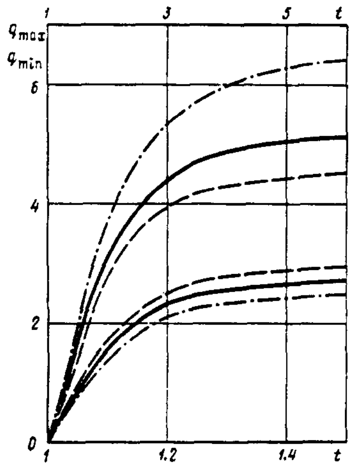

Fig.2

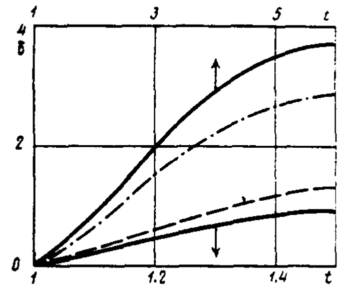

Fig.3

Fig.1 shows the contact stresses $q(r, t)$ as a function of r for $\mu = 10$, $t = 1.5$; $\mu = 0,1$, $t = 6$; $\mu = 1$ for the same two values of t. The latter case is shown in Fig.1 by a single solid line. This suggests that uniform aging of a packet does not affect the distribution of contact pressures under the stamp with a flat base. The stresses are independent of τ_1 and are determined by the function of the acting force $P(t)$, i.e. at the instants of time at which $P(t)$ has the same values, the distributions of the contact stresses are identical.

Fig.2 shows the changes in the maximum and minimum values of the contact stresses with time, depending on the character of the aging. The sold lines are correct in both time scales for the reasons already explained.

Fig.3 shows the subsidence under the stamp $\delta(t)$ as a function of time t for various cases of aging. The apperance of two solid lines implies the substantial influence of τ_1 on the subsidence $\delta(t)$ when $\mu = 1$.

The results given for the case of uniform aging can be obtained directly the correspondence principle /10/. Here they are discussed only in order to obtain a more complete picture of the phenomena, and for comparison purposes.

Let us state the basic conclusion arising from this point. In the case of natural non-uniform aging of a packet of layers the stress state under the stamp becomes more uniform with time, as compared with the case of uniform aging, i.e. from the point of view of stress concentration it improves with time. The subsidence is always greater in the case of natural non-uniform aging, than in the uniform case. Comparing the uniform and natural non-uniform aging of the foundation, we should note that the second case is characterized by a more non-uniform distribution (undesirable from the point of view of the concentration) of contact stresses and smaller values of the subsidence.

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Translated by L.K.

PMM U.S.S.R.,Vol.49,No.6,pp.782-786,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

CONTINUOUS THEORY OF DISLOCATIONS AND DISCLINATIONS IN A TWO-DIMENSIONAL MEDIUM*

YU.Z. POVSTENKO

A system of equations describing mobile defects in a two-dimensional Cosser at continuum, i.e. in a medium whose motion is determined by the displacement field and rotation field independent of it, is obtained.

The basic equations of the static theory /1—5/ and dynamic continuous theory /6—12/ of defects (dislocations and disclinations) are known for a three-dimensional medium, obtained by a variety of methods. A dislocation model of the misalignment surfaces used in describing the Martensitic transformations /2, 13/ is proposed. The dislocation representations were used in /14—16/ to describe the grain boundaries, and the difference dislocations within the boundaries of separation were studied in /17, 18/. The dislocation structure of internal boundaries of separation was described in /19, 20/ using the differential geometry characteristics (torsion and curvature tensors, non-holonomic object) of three-dimensional media. Surface dislocations and disclinations of the separate Volterra distortions-type were studied in /21/, with liquid crystals and various biological objects indicated as the suitable areas of application of these concepts.

1. Surface del operator. A surface imbedded in a three-dimensional Euclidean space is described by the equations $x^i = x^i(y^1, y^2)$ where y^α are curvilinear coordinates on the surface. Henceforth, the Latin indices will take the values of 1, 2, 3, and the Greek indices values of 1, 2. Regarding the radius vector $\mathbf{r}$ of a point on the surface as a function of the coordinates y^α, we introduce the local tangential basis vectors $\mathbf{a}_\alpha = \partial\mathbf{r}/\partial y^\alpha$ and the normal vector $\mathbf{n} = {}^1/_2\,\varepsilon^{\alpha\beta}\mathbf{a}_\alpha \times \mathbf{a}_\beta$ where $\varepsilon^{\alpha\beta}$ are the components of the Levi-Civita surface vector $\boldsymbol{\varepsilon}_\Sigma \doteq \varepsilon^{\alpha\beta}\mathbf{a}_\alpha\mathbf{a}_\beta$.

The surface del operator /22/

$$\nabla_\Sigma = \mathbf{a}^\alpha \partial/\partial y^\alpha$$

enables us to define, for the tensor $\mathbf{T}_\Sigma$ defined on the surface, the operations of surface grad, div and curl

$$\operatorname{grad}_\Sigma \mathbf{T}_\Sigma \equiv \nabla_\Sigma \mathbf{T}_\Sigma,\quad \operatorname{div}_\Sigma \mathbf{T}_\Sigma \equiv \nabla_\Sigma \cdot \mathbf{T}_\Sigma,\quad \operatorname{rot}_\Sigma \mathbf{T}_\Sigma \equiv \nabla_\Sigma \times \mathbf{T}_\Sigma$$

The rules of action of the surface del operator on the products of the quantities are identical to those of the three-dimensional del operator $\nabla = \mathbf{a}^k\partial/\partial x^k$ (see e.g. /23/. Essential differences due to the surface curvature appear on the second application of the two-dimensional del operator. For example, the following relations hold:

$$\nabla_\Sigma \times (\nabla_\Sigma \mathbf{T}_\Sigma) = \boldsymbol{\varepsilon}_\Sigma \cdot \mathbf{b} \cdot \nabla_\Sigma \mathbf{T}_\Sigma \tag{1.1}$$

$$\nabla_\Sigma \cdot (\nabla_\Sigma \times \mathbf{T}_\Sigma) = -2H\mathbf{n} \cdot (\nabla_\Sigma \times \mathbf{T}_\Sigma) + \nabla_\Sigma \cdot (\boldsymbol{\varepsilon}_\Sigma \cdot \mathbf{b} \cdot \mathbf{T}_\Sigma) \tag{1.2}$$

while in the three-dimensional case we have

$$\nabla \times (\nabla \mathbf{T}) = 0,\ \nabla \cdot (\nabla \times \mathbf{T}) = 0 \tag{1.3}$$

Here $\mathbf{b} = b_{\alpha\beta}\mathbf{a}^\alpha\mathbf{a}^\beta$ is the tensor of the second quadratic form of the surface and $H = {}^1/_2 b_\alpha{}^\alpha$. is the mean surface curvature.

2. Defects in the three-dimensional Cosserat continuum. To order to facilitate the presentation of the corresponding results for the two-dimensional Cosserat continuum, we shall give the basic equations for the three-dimensional medium (e.g. /24—26/).

The non-symmetric total deformation γ and flexure-torsion $\varkappa$ tensors are expressed in terms of the displacement $\mathbf{u}$ and rotation $\boldsymbol{\omega}$ vector thus

$$\boldsymbol{\gamma} = \nabla\mathbf{u} + \mathbf{g} \times \boldsymbol{\omega},\quad \boldsymbol{\varkappa} = \nabla\boldsymbol{\omega}$$